Formulae using day numbers

For stars and other objects outside the solar system the usual procedure for the computation of apparent positions from catalogue data is as follows, but the techniques described on pages B29–B31 should be used if full precision is required.

From	To	Step	Correction
catalogue epoch	current epoch	i	proper motion
catalogue equinox	mean equinox of year	ii	precession
mean equinox of year	mean equinox of date	iii	precession
mean equinox of date	true equinox of date	iv	nutation
true (heliocentric) position	apparent (geocentric) position	v	aberration (annual)
		vi	parallax (annual)

Star catalogues usually provide coefficients for steps i and ii for the reduction from catalogue position (α_0, δ_0) to the position for the mean equinox of another epoch. Besselian day numbers (A to E), which provide for steps iii to v for the reductions from the position (α_1, δ_1) for the mean equinox of the middle of the year to the apparent geocentric position (α, δ), are given on pages 4–11; for high declinations, the second-order day numbers (J, J') given on pages 12–15 may be required. The formulae to be used are:

$$\alpha = \alpha_1 + Aa + Bb + Cc + Dd + E + J \tan^2 \delta_1$$
$$\delta = \delta_1 + Aa' + Bb' + Cc' + Dd' + J' \tan \delta_1$$

where the Besselian star constants are given by:

$$a = (m/n) + \sin\alpha_1 \tan\delta_1 \qquad a' = \cos\alpha_1$$
$$b = \cos\alpha_1 \tan\delta_1 \qquad b' = -\sin\alpha_1$$
$$c = \cos\alpha_1 \sec\delta_1 \qquad c' = \tan\epsilon \cos\delta_1 - \sin\alpha_1 \sin\delta_1$$
$$d = \sin\alpha_1 \sec\delta_1 \qquad d' = \cos\alpha_1 \sin\delta_1$$

where α and δ are in arc units. For 2005·5, $m/n = 2{\cdot}301\,39$ and $\tan\epsilon = 0{\cdot}433\,54$.

The additional corrections for the proper motion (centennial components μ_α, μ_δ) during the fraction of year (τ) and for annual parallax (π) are given by:

$$\Delta\alpha = \tau\mu_\alpha/100 + \pi\,(dX - cY) \qquad \Delta\delta = \tau\mu_\delta/100 + \pi\,(d'X - c'Y)$$

where X, Y are the coordinates of the Earth with respect to the solar-system barycentre given on pages B34–B49. Strictly, this parallax correction should be computed using the coordinates of the Earth referred to the mean equinox of the middle of the year, or using star constants computed for the standard epoch of J2000·0.

The corrections for annual parallax may be included with the corrections for annual aberration by substituting $C - \pi Y$ for C and $D + \pi X$ for D in the formulae given above. Alternatively if the annual parallax is small enough it is possible to make the substitutions

$$c + 0{\cdot}0532\,d\pi \text{ for } c \qquad d - 0{\cdot}0448\,c\pi \text{ for } d$$
$$c' + 0{\cdot}0532\,d'\pi \text{ for } c' \qquad d' - 0{\cdot}0448\,c'\pi \text{ for } d'$$

The error in this approximate method is negligible if the parallax of the star is less than about 0″2.

A further correction to allow for the deflection of the light in the gravitational field of the Sun may also be required—appropriate formulae are given on page B17.

The day-number technique may also be used for objects within the solar system but steps i and vi are omitted and step v is replaced by forming the geocentric position by combining the barycentric position of the body at time $t - \Delta t$, where Δt is the light-time, with the barycentric position of the Earth at time t.

Example of day-number technique

To calculate the apparent place of a star at 0^h TT at Greenwich on 2005 January 1 from the mean place for J2005·5 using day numbers.

Step 1. From a fundamental star catalogue, such as the FK5, calculate for epoch and equinox J2005·5 the mean right ascension and declination (α_1, δ_1), the centennial proper motion (μ_α, μ_δ) and the parallax (π).

Assume the following fictitious values for the calculation:

$$\alpha_1 = 14^h\,39^m\,58^s\!.736 \qquad \delta_1 = -60^\circ\,51'\,27''\!.83 \qquad \pi = 0''\!.752$$
$$\mu_\alpha = -49^s\!.535 \text{ per century} \qquad \mu_\delta = +69''\!.42 \text{ per century}$$

Step 2. Form the star constants as follows:

$$a = \tfrac{1}{15}((m/n) + \sin\alpha_1 \tan\delta_1) = +0{\cdot}230\,27 \qquad a' = \cos\alpha_1 = -0{\cdot}766\,10$$
$$b = \tfrac{1}{15}\cos\alpha_1 \tan\delta_1 = +0{\cdot}091\,60 \qquad b' = -\sin\alpha_1 = +0{\cdot}642\,72$$
$$c = \tfrac{1}{15}\cos\alpha_1 \sec\delta_1 = -0{\cdot}104\,88 \qquad c' = \tan\epsilon\cos\delta_1 - \sin\alpha_1\sin\delta_1 = -0{\cdot}350\,23$$
$$d = \tfrac{1}{15}\sin\alpha_1 \sec\delta_1 = -0{\cdot}087\,99 \qquad d' = \cos\alpha_1 \sin\delta_1 = +0{\cdot}669\,12$$

Step 3. Extract the day numbers from pages B24, B34 and B35. In general, linear interpolation is required and second differences may be significant for A and B. The values for 2005 January 1 at 0^h TT are:

$$A = -12''\!.956 \qquad C = -3''\!.541 \qquad E = -0^s\!.0010 \qquad J = +0^s\!.000\,18$$
$$B = -7''\!.596 \qquad D = +20''\!.470 \qquad \tau = -0{\cdot}4993 \qquad J' = -0''\!.0015$$

Step 4. Extract the values of the Earth's rectangular coordinates from page B44 (the values for J2000·0 are of sufficient accuracy for computing the parallax correction). The values are:

$$X = -0{\cdot}177 \qquad Y = +0{\cdot}887$$

Step 5. Calculate the corrections for light-deflection, $\Delta\alpha$ and $\Delta\delta$.

For the Sun for 2005 January 1 at 0^h TT, $\alpha_0 = 18^h\,46^m\!.4$, $\delta_0 = -23^\circ\,01'$. Using the formulae on page B17, cos (elongation) = +0·5546 and the corrections for light-deflection are $\Delta\alpha = -0^s\!.001$ and $\Delta\delta = 0''\!.00$.

Step 6. Compute the apparent position as follows:

Mean position 2005·5,	$\alpha_1 = 14^h\,39^m\,58^s\!.736$		$\delta_1 = -60^\circ\,51'\,27''\!.83$
$Aa + Bb + Cc + Dd + E$	$= -5^s\!.110$	$Aa' + Bb' + Cc' + Dd'$	$= +19''\!.98$
$J\tan^2\delta_1$	$= +0^s\!.001$	$J'\tan\delta_1$	$= mwd;\ 0''\!.00$
$\tau\mu_\alpha/100$	$= +0^s\!.247$	$\tau\mu_\delta/100$	$= -\ 0''\!.35$
$\pi(dX - cY)$	$= +0^s\!.082$	$\pi(d'X - c'Y)$	$= +\ 0''\!.14$
$\Delta\alpha$	$= -0^s\!.001$	$\Delta\delta$	$= mwd;\ 0''\!.00$
Apparent position	$\alpha = 14^h\,39^m\,53^s\!.954$		$\delta = -60^\circ\,51'\,08''\!.05$